

GAUGE FIELDS ON TORUS AND PARTITION FUNCTION OF STRINGS

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International Journal of Modern Physics **A4** (1989) pp. 389–400

Abstract

In this paper we consider the interrelation between compactified string theories on torus and gauge fields on it. We start from open string theories with background gauge fields and derive partition functions by path integral. Since the effects of background fields and compactification correlate only through string zero modes, we investigate these zero modes. From this point of view, we discuss the Wilson loop mechanism at finite temperature. For the closed string, only a few comments are mentioned.

1 Introduction

It is believed that the string theory is one of the promising candidates for the unified theory.[1] String theories have many different ingredients than field theories. Firstly, in string theories, there exist infinitely many heavy particles as string oscillation modes. The existence of these particles modifies the quantum behavior of the theory. Secondly, since string theories are often constructed in higher dimensions, the compactification of extra dimensions is required. As a result, heavy particles appear in inverse proportion to the size of the internal space. For they have mass scales about Planck mass, it cannot seem to produce them directly by the present accelerators.

It can be thought that we will find indirectly, these characteristic phenomena of string theories only in cosmology; especially in the early universe at high temperature state, there is a possibility that even the space-time structure is different from now. At present, it might be able to observe the traces and influences of the early stage of the universe.

When we consider these possibilities, it is necessary to investigate the relation between the string theory and the compactification of extra spaces. Further, since string theories contain many kinds of fields including gauge fields, we have to examine their own roles.

In this paper we consider the gauge field on torus and its correlation with string zero-mode components. Then we investigate the (one-loop) partition function described as path integral in order to understand how to relate the mass spectra of matter fields with gauge fields.

One of the motivations of this work is to apply the Kaluza-Klein ideas to string theories. Here, the treatise of background gauge fields and/or compactified spaces require almost the same prescription as taking appropriate assumptions on background gauge fields when the dimensional reduction is done in Kaluza-Klein theory. We will find “Kaluza-Klein” excitation mode on each string oscillation mode spectrum.

Another motivation comes from the interest in the problem of gauge symmetry breaking in the string theory. String theories formulated in higher dimensions are generally required to have large gauge symmetries. The idea to derive almost realistic gauge symmetry when the space is compactified is already used in Kaluza-Klein field theory. In this mechanism [2] gauge fields as the background field have an important role; in the string theory, especially its path-integral approach, it is interesting to know how that mechanism realizes. In addition, whether “symmetry breaking” occurs as “phase transition” or not, and what is the difference from the Higgs mechanism, are also interesting topics. For the sake of these, it is necessary to compute the thermo-partition function. In path-integral formalism, both for field theories and for string theories, the use of “imaginary-time method”, [3] which is known for introducing temperature, simplifies the derivation of the thermo-partition function. Therefore it seems that important results can be found clearly by considering background gauge fields in string path integral.

The contents of this paper are as follows. In Sec. 2, we consider the partition function of open strings with a magnetic flux on two-dimensional torus. Although the result is not a new one, the derivation is considerably simplified by the path-integral approach. In Sec. 3, we investigate the partition function with gauge fields of nonzero value on torus. This is the case known as Wilson-loop or Hosotani mechanism, [2] and connected with the spontaneously symmetry breaking. The dependence of the partition function on temperature is discussed. In Sec. 4, some comments are given for closed strings and gauge fields. Sec. 5 is devoted to summary and outlooks.

2 Magnetic Field and Open String on Two Dimensional Torus

In this and next sections, we study the open string theory. In this section, a $U(1)$ gauge field is considered, while non-Abelian gauge field is studied in the

next section.

When we take into account the background electromagnetic field on bosonic open string theory, the action of two-dimensional world sheet is given as follows:[4, 5]

$$S = \frac{T}{2} \int d^2\sigma \sqrt{-g} g^{ab} \partial_a X^M \partial_b X_M + iT \oint ds \partial_s X^M A_M, \quad (1)$$

where X 's are (26-dimensional) bosonic space-time coordinates, A_M is a background electromagnetic field. T is the string tension.

In this section, we consider the partition function of open strings with a constant magnetic flux B on two-dimensional torus. Here, it is assumed that the partition function has already been found in the case with flat space-time and vanishing magnetic field. Namely, we would like to give attention to the effect of compactifications and the magnetic field on the partition function through the string zero-mode.

We restrict ourselves to the case of the “neutral string” referred in Ref. [4]. Its zero-slope limit yields a Maxwell (electromagnetic) theory.

We consider the mode expansion of string coordinates, it is known that, by the effect of the compactifications or the presence of the magnetic field, the eigenfunctions of the oscillator part are changed only by phase,[4] while their eigenvalues, which give the mass spectrum, is never changed. So, we give attention to only zero-mode piece which concerns with “Kaluza-Klein” excitation states.

The shape of the world sheet corresponding to the one-loop calculation is a cylinder-like surface. We assume the path along the σ_2 coordinate is closed, and the metric is given as follows:[6]

$$ds^2 = d\sigma_1^2 + t^2 d\sigma_2^2. \quad (0 \leq \sigma_1, \sigma_2 \leq 1) \quad (2)$$

(t is called the moduli parameter.)

Let X^1 and X^2 be the torus coordinates, and let r_1 and r_2 be their radius respectively. The zero mode parts of them are assumed to satisfy the following condition,

$$\begin{aligned} X^1(\sigma_1, \sigma_2 + 1) &= X^1(\sigma_1, \sigma_2) + 2\pi r_1 l, \\ X^2(\sigma_1, \sigma_2 + 1) &= X^2(\sigma_1, \sigma_2) + 2\pi r_2 m. \quad (l \text{ and } m \text{ are integers}) \end{aligned} \quad (3)$$

These conditions indicate the fact that points separated by $2\pi r_{1,2}$ units in each direction are identified with each other, therefore they represent a torus.

Let the following magnetic field exists on the torus,

$$F_{12} = -F_{21} = \nabla_1 A_2 - \nabla_2 A_1 \equiv B. \quad (4)$$

Since this field is a kind of “monopole” magnetic fields, B is subject to a quantization condition.[7] However, as seen later, a conclusion can be derived without that condition.

From the action (1), it is understood that the following boundary conditions have to be satisfied,[4]

$$t\partial_{\sigma_1}X^1 = iB\partial_{\sigma_2}X^2, \quad t\partial_{\sigma_1}X^2 = -iB\partial_{\sigma_2}X^1. \quad (5)$$

From Eqs. (3) and (5), the zero mode components can be written as follows:

$$\bar{X}^1 = \frac{iB}{t}2\pi r_2 m\sigma_1 + 2\pi r_1 l\sigma_2, \quad \bar{X}^2 = -\frac{iB}{t}2\pi r_1 l\sigma_1 + 2\pi r_2 m\sigma_2. \quad (6)$$

Next, let us see the part including gauge fields in the action. The line integral of the gauge field along the boundary can be re-written by the surface integral of the field strength on the world sheet, i.e.,

$$iT \oint ds \partial_s X^M A_M = -\frac{i}{4} \int d^2\sigma \varepsilon^{ab} \partial_a X^M \partial_b X^N F_{MN}. \quad (7)$$

Now, we can find the contribution of zero-mode pieces in the partition function. From (2.1), (2.2), (2.6) and (2.7), we find that the partition function is proportional to the following factor;

$$\begin{aligned} & \sum_{l=-\infty}^{\infty} \sum_{m=-\infty}^{\infty} e^{-\bar{S}} \\ &= \sum_l \sum_m \exp \left[-T \left\{ \frac{1}{2} \int d^2\sigma \left(t\partial_{\sigma_1} \bar{X}^M \partial_{\sigma_1} \bar{X}_M + \frac{1}{t} \partial_{\sigma_2} \bar{X}^M \partial_{\sigma_2} \bar{X}_M \right) \right. \right. \\ & \quad \left. \left. - \frac{i}{4} \int d^2\sigma \varepsilon^{ab} \partial_a \bar{X}^M \partial_b \bar{X}^N F_{MN} \right\} \right] \\ &= \sum_l \sum_m \exp \left[-T \left\{ \frac{1}{2} t \frac{(-B^2)}{t^2} (2\pi r_2 m)^2 + \frac{1}{2t} (2\pi r_1 l)^2 \right. \right. \\ & \quad \left. \left. + \frac{1}{2t} \frac{(-B^2)}{t^2} (2\pi r_1 l)^2 + \frac{1}{2t} (2\pi r_2 m)^2 + \frac{B^2}{2t} [(2\pi r_1 l)^2 + (2\pi r_2 m)^2] \right\} \right] \\ &= \sum_l \sum_m \exp \left[-\frac{T}{2t} \{ (2\pi r_1 l)^2 + (2\pi r_2 m)^2 \} \right]. \end{aligned} \quad (8)$$

Then, it turns out to be independent of the magnetic field B . The partition function including the contribution from oscillators is then given by

$$Z \propto \int_0^\infty \frac{dt}{t} t^{-13} e^{\pi t} \left[\prod_{n=1}^{\infty} (1 - e^{-\pi n t}) \right]^{-24} \sum_{l,m} \exp \left[-\frac{T}{2t} \{ (2\pi r_1 l)^2 + (2\pi r_2 m)^2 \} \right], \quad (9)$$

where t plays a role of the Schwinger parameter of the heat-kernel method.[8] By using Jacobi's imaginary transformation,[9] we can find the particle mass spectrum; namely, the partition function is represented by the integration over t (the heat-kernel integral), and can be written as the following form,

$$Z \sim \sum_i \int_0^\infty \frac{dt}{t} t^{-D/2} \exp(-tM_i^2), \quad (D \text{ is the dimension of the spacetime}) \quad (10)$$

where, M 's stand for masses of the particles, Σ symbolically represents the summation over the particle species including the degeneracy. As a result, using Jacobi's imaginary transformation, we get

$$\sum_l \exp \left[-\frac{T}{2t} (2\pi r l)^2 \right] = \sqrt{\frac{t}{2\pi T r^2}} \sum_l \exp \left[-\pi t \frac{l^2}{2\pi T r^2} \right]. \quad (11)$$

Thus it can be read from the form of the partition function that the mass spectrum is as follows;

$$M^2 = 2\pi T N' + \frac{l^2}{r_1^2} + \frac{m^2}{r_2^2}, \quad (l \text{ and } m \text{ are integers}) \quad (12)$$

where N' is the occupation number of oscillators.

Being independent of whether the magnetic field exists or not, the mass spectrum has a structure that each string-oscillation mode contains Kaluza-Klein excitation modes. Because we study the “neutral string” called in Ref. [4] in this section, the fact that the spectrum is independent of magnetic field is physically rather trivial. It is interesting, however, that the contribution of string zero-mode under the particular boundary condition and that from the part of the action including gauge field are cancelled by each other in the partition function, it is independent of the quantization condition of the magnetic flux on the compactified space.

In the next section, we will consider the case that the field strength is zero while there exist nontrivial gauge fields on torus.

3 Open String Theory and Hosotani Mechanism

As mentioned in Sec. 1, the method of breaking the gauge symmetries is an important problem which arises commonly in any unified theories. In this section, we study the change of particle spectrum by Wilson loop mechanism (Hosotani mechanism [2]) with the torus compactification in open string theories. In addition, we introduce temperature and discuss the possibility of the phase transition with temperature.

In this section, we consider open superstring theories which contains $SO(N)$ gauge symmetry. Supersymmetric string action can be expressed by various forms. Here, we take the covariant one, for example, as in Ref. [10] (of open string version). Since we are now concerned with the part directly connecting gauge fields and zero modes of bosonic coordinate, we do not write down the total action. The difference between the part of the action including gauge fields and the previous one (1) is that the gauge field is represented as $N \times N$ matrix in the present case. When one calculates the partition function, the following factor appears:[11]

$$\prod_{boundaries} \text{Tr} \exp \left[i \oint ds \partial_s X^M \mathbf{A}_M \right]. \quad (13)$$

This form exactly represents the sum of the (gauge-invariant) Wilson-loop elements. In case of $SO(N)$ symmetry action, string one-loop diagrams have not only a cylinder but also a twisted band—Möbius band. Now, we restrict ourselves to consider the most simple assumption on the gauge field. Namely, the gauge field for the direction X^I of the torus coordinate is given by

$$\mathbf{A}_I = A \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}. \quad (14)$$

Here, in general, it is expected that the symmetry breaking like $SO(N) \rightarrow SO(N-2) \times U(1)$ occurs. The zero-mode part of the coordinate can be written by, as the previous section,

$$\bar{X}^I = 2\pi r l \sigma_2. \quad (l \text{ is an integer.}) \quad (15)$$

Substituting (14) and (15) into (13), one can get the following factor for each diagram.[11]

$$\begin{aligned} \text{For the cylinder : } & (N + 2\{\cos(2\pi l\phi) - 1\})^2, \\ \text{for the Möbius strip : } & N + 2\{\cos(4\pi l\phi) - 1\}, \end{aligned} \quad (16)$$

where $\phi = rTA$.

Here, in the trivial case, i.e. with $\phi = 0$, we ought to note that the factors for a cylinder and Möbius strip become N^2 and N respectively. The partition function is expressed as the summation of contributions from the cylindrical and Möbius band diagram, and these factors agree with the results derived from Chan-Paton factors.[12]

Now, the partition function is, even with the compactification, identically zero because of supersymmetry. So, here we consider the finite temperature case for an interesting example. The introduction of temperature has already been achieved by many people.[10, 13] The method can be seen as almost the same as the “imaginary time method”[3] in the field theory. By making the zero-mode part in the time direction be periodic, we introduce the temperature into the path integral for strings. Namely,

$$\bar{X}^0 = \beta n \sigma_2, \quad (n \text{ is an integer}) \quad (17)$$

where β means the inverse of temperature.

For fermions, we may use the formalism developed in Ref. [10]. For the boundary conditions of fermionic coordinates, we take only the following one corresponding to n in (17):

$$\theta(\sigma_1, \sigma_2 + 1) = (-1)^n \theta(\sigma_1, \sigma_2). \quad (18)$$

This yields just the imaginary time method in superstring version, that is, “fermion fields change their sign when they are pushed by a period for the closed

time direction". The derivation of the partition function can be carried over in parallel with Ref. [11]. Let us refer to it for the calculations.

As a final procedure, we sum up two contributions from one-loop diagrams, taking into account their relative sign. The sign can be determined from the requirement of existence of $SO(N)$ gauge fields at zero-mass level, for satisfying unitarity. Then, we obtain the following result.

$$\begin{aligned}
F &\propto -16T^5 \int_0^\infty \frac{dt}{t} (2\pi t)^{-5} \sum_{n=0}^\infty \exp \left[-\frac{T}{2} \frac{\beta^2}{t} (2n+1)^2 \right] \\
&\times \sum_{l=-\infty}^\infty \exp \left[-\frac{T}{2} \frac{(2\pi r)^2}{t} l^2 \right] \cdot \frac{1}{2} \left[\{N + 2(\cos(2\pi\phi l) - 1)\}^2 \prod_{p=1}^\infty \left(\frac{1 + e^{-\pi p t}}{1 - e^{-\pi p t}} \right)^8 \right. \\
&\quad \left. - \{N + 2(\cos(4\pi\phi l) - 1)\} \prod_{p=1}^\infty \left(\frac{1 + (-e^{-\pi t})^p}{1 - (-e^{-\pi t})^p} \right)^8 \right]. \quad (19)
\end{aligned}$$

From this partition function, we can find the mass spectrum of the particles by using Jacobi's transformation as seen in Sec. 2. This is a little more complicated than the previous one;

$$\sum_{l=-\infty}^\infty \exp \left[-\frac{(2\pi r)^2 T l^2}{2t} \right] \cos(2\pi\phi l) = \sqrt{\frac{t}{2\pi T r^2}} \sum_{l=-\infty}^\infty \exp \left[-\pi t \frac{(l - \phi)^2}{2\pi T r^2} \right]. \quad (20)$$

This indicates that Kaluza-Klein excitation modes with the difference by the gauge field contribution.[14] In the field theory, when there exist gauge fields on torus, the free energy can be written formally as follows;

$$F \propto \frac{1}{2} \text{Tr} \ln [D_M^2 + M^2]. \quad (21)$$

Here, since covariant derivatives contain gauge fields, we may understand that the discrete excitation levels on torus originated from these derivatives are shifted by the contribution from vacuum gauge fields. It is interesting that this local ("particle-like") viewpoint connects with a global ("string-like") one by the transformation (20). By the way, we note here that the concrete expression of the free energy agrees exactly with (19) (see Ref. [13] etc.). In fact, applying Eq. (21) into (20), we see the ϕ dependence of mass spectrum;

for even N' :

$$M^2 = \begin{cases} 2\pi T N' + l^2/r^2 & \text{with degeneracy } (N-2)(N-3)/2 + 1 \\ 2\pi T N' + (l - \phi)^2/r^2 & \text{with degeneracy } 2(N-2), \end{cases}$$

for odd N' :

$$M^2 = \begin{cases} 2\pi T N' + l^2/r^2 & \text{with degeneracy } (N-2)(N-1)/2 + 1 \\ 2\pi T N' + (l - \phi)^2/r^2 & \text{with degeneracy } 2(N-2) \\ 2\pi T N' + (l - 2\phi)^2/r^2 & \text{with degeneracy } 2. \end{cases} \quad (22)$$

As above, the patterns of the particle mass spectra modified by the expectation value of the gauge fields exactly indicate the adjoint or symmetric representations of $SO(N)$, corresponding to even or odd number of string oscillators

respectively. From other perspective, in particle-like viewpoint, it corresponds to the fact that in Eq. (21), the covariant derivatives are as follows:

If λ_A is in the adjoint representation :

$$D_M \lambda_A \sim \partial_M \lambda_A + i[\mathbf{A}_M, \lambda_A].$$

If λ_S is in the symmetric representation :

$$D_M \lambda_S \sim \partial_M \lambda_S + i\{\mathbf{A}_M, \lambda_S\}. \quad (23)$$

It is interesting that the factor given by the sum of two different world sheet diagrams, cylinder and Möbius band, turns out to give correctly the symmetric patterns of the group.

The “free energy” (19) may be regarded as a potential of ϕ . Now we proceed in this line.

We find that $\phi = 0$ (modulo 1) is the minimum of the free energy even at nonzero temperature (of course, at zero temperature it closes to be identically zero) in this model. Thus we cannot have naive image of phase transitions from this model. Unfortunately, the same may be true whenever we consider the free energy with the Wilson-loop on torus. Before replacing the model with more complicated ones, let us investigate its temperature dependence in order to get a physical insight.

First of all, we note that when temperature (β^{-1}) is extremely smaller than the square root of the string tension ($T^{1/2}$), the behavior of the free energy agrees with that obtained by usual field theoretical techniques because we can neglect the excitation of string oscillations. Similar to Ref. [14], the potential may be regarded as a summation of the form $\cos(2\pi l\phi)$, so we can only think about its coefficient for each l . In short, it is important that the cosine functions do not contain the parameter t of the integration.

To proceed in detail, in the model with supersymmetry as considered here, the analysis of the potential at low temperature is difficult because of the cancellation of the coefficients which assume large contributions in the potential. Without a numerical calculation, we investigate it by the following method. Firstly, the extremum of the potential can be at $\phi = 0$ or $1/2$, because the potential is given by the sum of trigonometric functions. Next, we should consider the difference of the potential at $\phi = 0$ and $\phi = 1/2$. The difference is reduced to the following summation,

$$-\sum_{l:\text{odd}} \sum_{n:\text{odd}} \frac{1}{[(2\pi r l)^2 + (\beta n)^2]^5}. \quad (24)$$

As far as the string excitation modes can be neglected, the difference of the potentials is proportional to this summation. Trivially, (24) is negative, so it is expected that at low temperature ($\beta^{-1} \ll T^{1/2}$) the minimum of the free energy is realized when $\phi = 0$. Especially in the case of $\beta^{-1} \ll r^{-1}$, the difference of the free energy at $\phi = 0$ and $\phi = 1/2$ behaves like β^{-9} .

Next, we consider the high temperature case. In the string theory, there exists the critical temperature [15] at which several physical quantities diverge.

But the difference of the potential as studied above remains finite. At the critical temperature, the difference of the energy at $\phi = 0$ and $\phi = 1/2$ can be evaluated as follows,

$$\begin{aligned} \Delta F &\equiv F(\phi = 0) - F(\phi = 1/2) \\ &\sim -(N-2)(2\pi)^{-5}T^4 \sum_{l=0}^{\infty} \sum_{n=0}^{\infty} \frac{1}{(2\pi r)^2(2l+1)^2 + 4\beta_c^2 n(n+1)}, \end{aligned} \quad (25)$$

where $\beta_c = (4\pi/T)^{1/2}$ is the inverse of the critical temperature.

Therefore, we can find that even if string oscillations come into the calculation, the locations of the minima of the potential (free energy) are never shifted at high temperature.

From the brief investigation given here, we can guess that the minimum of the potential is independent of the temperature in general models. Further, even at high temperature, since the critical temperature is the order of the lowest string excitation mode, we will understand that at “medium” temperature the “correction” by string oscillation gives little influence.

It may be conceivable to see other mechanism that is more complicated but possible to break gauge symmetry for instance compactifications on orbifolds etc.; to study them will be a future problem.

4 Comments on Closed Strings

The partition function for closed strings on torus is definitely given by Sakai and Senda.[16] They have used Hamiltonian (canonical) formalism; however the same result can be obtained by the path integral method. In this method, we impose a Kaluza-Klein-like assumption on the space-time metric G_{MN} ; then, we can obtain half of the zero-mode gauge fields, corresponding to the Cartan subgroup of the gauge group generated by compactification, while the rest is included by consideration of the antisymmetric tensor B_{MN} . It is symbolically written as follows.

Gauge fields of zero modes A and B corresponding to the Cartan subgroup $H \times H'$ of the gauge field $G \times G'$ are given by

$$A_\mu \sim G_{\mu I} \quad \text{and} \quad B_\mu \sim B_{\mu I}, \quad (26)$$

where I 's are the indices of torus-compactified dimensions.

These representations can be used to introduce chemical potential.[17]

Now, we may consider Wilson-loops in terms of these gauge fields. As a concrete example, we will investigate bosonic strings in twenty-six dimensions and the case that one dimension is compactified on torus. When the size of the torus takes a special value, we can regard it as the theory possessing $SU(2) \times SU(2)$ gauge symmetry in twenty-five dimensions.[16] And then letting another dimension be torus, we can set A and B have nonzero values on it. This formulation leads to the nontrivial Wilson-loop elements.

Regarding it as an original twenty-six dimensional theory, the existence of nonzero A and B is equivalent to the possibility of two-dimensional torus to take various forms and nonzero values of antisymmetric tensor fields on it. As shown by Sakai and Senda, with the metric and antisymmetric tensor fields of special values, the gauge symmetry can be extended to $SU(3) \times SU(3)$.

Considering in twenty-five dimension again, it can be said, “the gauge symmetry is extended’ by the Wilson-loop mechanism.[11] Of course, even by taking into account the Kaluza-Klein excitation of gauge fields, it never occurs in usual field theories.

It was not shown explicitly by Sakai and Senda, however, that the existence of antisymmetric tensor fields is a necessary condition for what is shown above to occur. However a further essential point will be that, in the closed string theory, quantum numbers are restricted by the freedom in reparametrization of string coordinates. The constraint can be, although not written here, derived from path integral formalism of partition function. (See Ref. [13].) We should note that the constraint on quantum numbers is never changed with any values of A , B , and the radius of the torus. We must give attention to the fact that when we introduce temperature by the imaginary time method, the time axis can be regarded as torus while there is no restriction by coordinate reparametrization in its direction.

This restriction and the existence of tachyon are necessary conditions for the mechanism of gauge-symmetry extension in the closed string theory. In the heterotic string theory, since gauge fields can be seen as obtained from compactification, it must be careful when we consider Wilson-loop (Hosotani) mechanism. The practical calculation of the free energy with constant gauge fields is straightforward in the path integral formalism, because many authors have considered such background fields in the context of the study in the string theory with various gauge symmetries.[18]

Consequently, when we are concerned with the Wilson-loop or vacuum gauge fields, in case of the open string the particle viewpoint remains true, since the interaction with external gauge fields is restricted at the edges of world sheet; meanwhile in the closed string theory there appears essentially characteristic property of the string theory. However, this is the case in one loop level. Even in the open string theory the ‘stringy’ nature will appear in higher loop levels; the corresponding diagrams might be the form containing closed strings in those levels.

In the closed string theory, we must take care of the winding soliton states. Thus the evaluation of the free energy becomes difficult when the scale of compactification is small. We suppose the treatment of the Wilson-loop mechanism can be easy in the “canonical” way that, roughly speaking, we construct explicitly mass and charge eigenstates. The line of the thought will be developed in separate publications.

5 Summary

In this paper, we studied how string partition function is affected by the part containing gauge fields on torus and string zero mode in the path integral formalism. Especially, because the size of the torus and the temperature are represented by zero mode of strings, the partition function can be given through a simple derivation.

In future, we would like to consider whether the “Kaluza-Klein” idea is applicable to even the case fully affected by characteristic properties in string theory, such as, the closed string theory on orbifolds and Wilson-loops on it [19] and the superstring theory in lower dimensions than ten.[20]

As for the dependence of the partition function on temperature, it will be interesting to consider compactification of nonsupersymmetric string theories.[21] In these kind of theories it is known, for example, that the partition function at zero temperature, i.e. the cosmological constant, asymptotically goes to zero with the radius of compactified torus.[22] Such models will be expected to lead a new interest about cosmological evolution of the space-time structure and (gauge) symmetries. These models are described by the closed string theories and have some complexities; nevertheless, when we consider the early universe, it should be required to search for the partition function at finite temperature and to investigate its behavior.

Finally, it must be mentioned the development of four-dimensional string theories.[23] However, the theories with extra dimensions should be investigated with sufficient interest, because of (1) the possibility to solve the problems in (four-dimensional) cosmology,[24] (2) the requests from quantum cosmological models,[25] (3) to supply a mechanism for supersymmetry breaking,[26] and so on; further they might have many more important roles.

Acknowledgments

We thank T. Hori for reading this manuscript. One of us (K.S.) thanks Iwanami Fūjukai for financial support.

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